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XIII.

CONTRIBUTIONS FROM THE PHYSICAL LABORATORY OF THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

XLII. — ON AN APPARATUS FOR THE MEASUREMENT OF COEFFICIENTS OF SELF-INDUCTION, AND THE INVESTIGATION OF THE PHENOMENA OF ALTERNATING CURRENTS.

By Frank A. Laws.

Presented by Charles R. Cross, January 10, 1894.

The increasing use of alternating currents in telephony and electric lighting has, during the last few years, directed attention to the measurement of coefficients of self-induction. In this paper certain apparatus devised for this purpose, together with methods for its use, will be described. Further discussion of the various methods of measurement proposed and of experimental results obtained is reserved for a later article.

For investigations pertaining to telephony it is particularly desirable to measure inductances by means of alternating currents of suitable known frequencies, and of the same order of magnitude as those used in practice. In all such work the reduction of the results is greatly simplified if the E. M. F. employed follows the sinusoidal law.

Therefore the aim at the outset was to produce a dynamo whose E. M. F. should vary in this manner, and whose frequency of alternation should be about that which is normal in telephonic work. Under these requirements there have been designed in the Rogers Laboratory two dynamos, or arrangements of dynamos. The first of these was planned during the winter 1890–91, and was used by Mr. L. Derr in connection with his thesis in 1891–92. This machine was provided with a stationary armature having Lord Kelvin's zigzag winding on the outer surface of a hollow cylinder. This was placed in the gap between a star-shaped piece of iron and a disk with a corresponding star-shaped opening, the two pieces being similarly and concentrically placed and forming the pole pieces of the machine. Owing to the shape of the pole pieces the field between them is variable, being denser at the points of the stars, so that when the field moves past the armature an E. M. F. will be set up in the latter.

One disadvantage of this design is that the E. M. F. is small, owing to the very large non-effective field; for it is only the variation of the field that is of service in producing the E. M. F. By properly shaping the pole pieces a curve of E. M. F. was obtained, which, when plotted, was found to be nearly sinusoidal in form.

The second machine was constructed by Messrs. C. L. Norton and P. H. Thomas, of the class of 1893, and forms the subject of the present paper. The special form of dynamo used was designed in accordance with a suggestion from them. The frequency is somewhat low, being 400 complete alternations per second. When another machine is built, it will be so designed that the superior limit will be raised to 2,000 complete periods per second.

Both the machine of 1890 and that of 1892 embody the same general features. Each has two independent armatures, one fixed, the other movable in phase relation to the first. This relation is capable of measurement. At the outset it was determined that in these machines the armatures should contain no iron, and that they should remain stationary, in order to eliminate any possible microphonic action at the brushes.

The machine, which is shown in place ready for use in Figure 1, will now be described. It consists, in brief, of two dynamos, with revolving fields mounted on one shaft to insure definite phase relation. The armature of one is fixed, while that of the other may be given any desired angular advance or retardation in reference to the first by means of a tangent screw. This advance can be read off from the graduated head of the screw. The fields are independent of one another. In addition to the dynamos the shaft carries a contact arrangement for mapping wave-forms, and a stroboscopic disk for determining the speed of rotation.

One of the field magnets is shown in plan in Figure 3 and in section in Figure 2. It consists of two star-shaped pieces of cast iron, keyed to the shaft and having hubs of such a length that the faces of the teeth are $\frac{5}{16}''$ apart, thus leaving a gap in which is the armature, $\frac{1}{4}''$ thick. The castings are recessed to receive the field coils, which are about one square inch in section and contain 250 turns of No. 16 B. & S. double cotton-covered wire. The coils were wound on forms, paraffined and taped, and were inserted when the machine was assembled. The terminals are brought out through bushed holes and

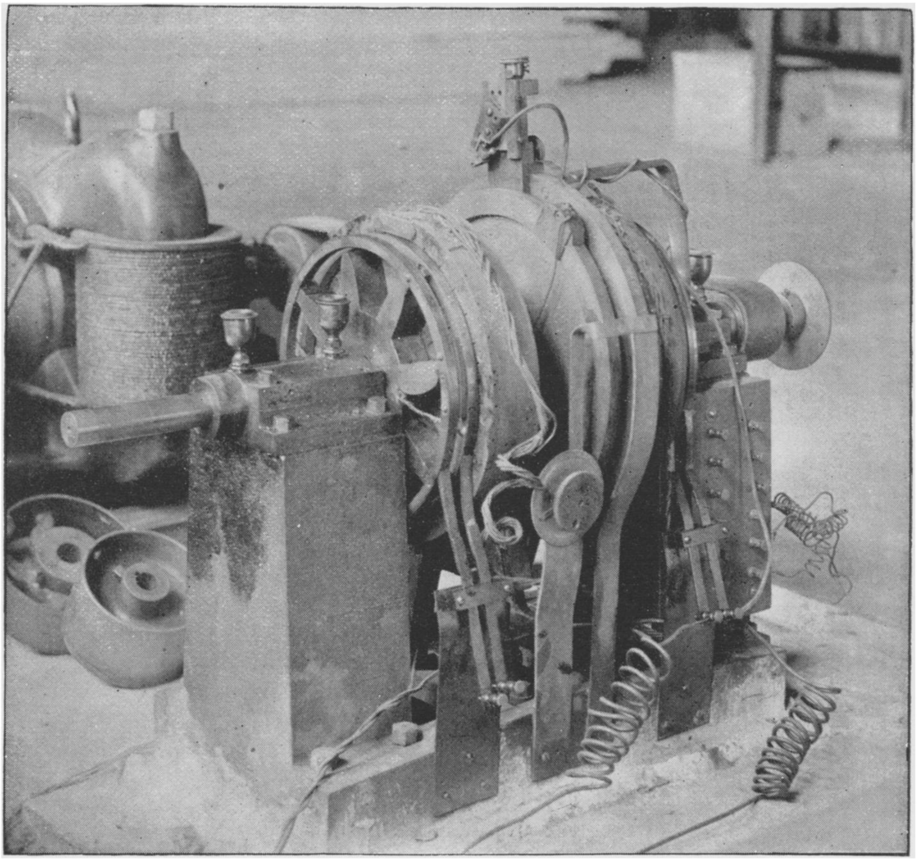

Fig. 1.

fastened to composition collector rings, which are carried and insulated by blocks of vulcanite mounted on the tips of the teeth of one of the castings. The brushes of spring copper are supported from the base plate. The eight teeth were milled from the round by a special Brown and Sharpe cutter, and have the form of a sinusoid laid off radially from the base circle. Of course, owing to the spreading of the lines of force, the particular form to be given to the teeth in order to obtain the best action can only be ascertained by trial.

The armature consists of a vulcanite disk, $10\frac{1}{4}''$ diameter, $\frac{1}{4}''$ thick, having a $4''$ hole in the middle. From this hole radiate sixteen equally spaced saw-cuts $\frac{3}{16}''$ deep, $\frac{1}{16}''$ wide, and $2\frac{1}{4}''$ long, the outer extremities of alternate pairs of cuts being connected by grooves concentric with the disk. In these cuts and grooves the armature wires are laid in shellac. The windings at present consist of three sections having respectively 10, 10, and 20 turns of No. 30 B. & S. copper wire. The total resistance is about ten ohms. The manner of winding as well as the action of the dynamo, by varying the length of active wire, are evident from Figure 3. The armatures are carried by built up hard-wood rings, $10\frac{1}{4}''$ outside diameter, $8\frac{1}{4}''$ inside, and $1\frac{9}{16}''$ deep. The ring in one dynamo is attached to a brass disk and hub, which fit the shaft and are rigidly bolted to the frame of the machine.

In the other dynamo, the armature, ring, disk, and hub are attached to a worm-wheel, and the whole is movable on a sleeve supported firmly from the frame of the machine. Through this sleeve passes the shaft without contact. The worm-wheel gears into a tangent screw provided with a divided head, the gearing being such that 1° on the wave form is represented by 20° on the head. Backlash is prevented by a spiral spring. See Figures 2 and 3.

Between the two dynamos is a brass disk, keyed to the shaft and provided with an insulated collector ring and contact point. By the side of this disk is a radial arm, revolving freely about the shaft, and capable of being clamped at any desired point to a fixed graduated circle. The arm carries the contact spring, made of watch spring and supported on its lower side very near its extremity to prevent the lengthening of the arc of contact by vibration. The spring can be moved out of contact with the point by a fine-pitched screw. This contact arrangement we have found to work satisfactorily at as many as 3,000 revolutions per minute.

The shaft of the machine is of steel, $1''$ in diameter, $29''$ long, and runs in composition bearings, each of which is $5\frac{1}{2}''$ long. The frame of the machine is heavy, weighing 175 pounds, and is designed so as to be very rigid. Driving is effected by means of a 2″ belt running over a 5″ pulley. The motor (1 H. P.) is of the Sprague type.

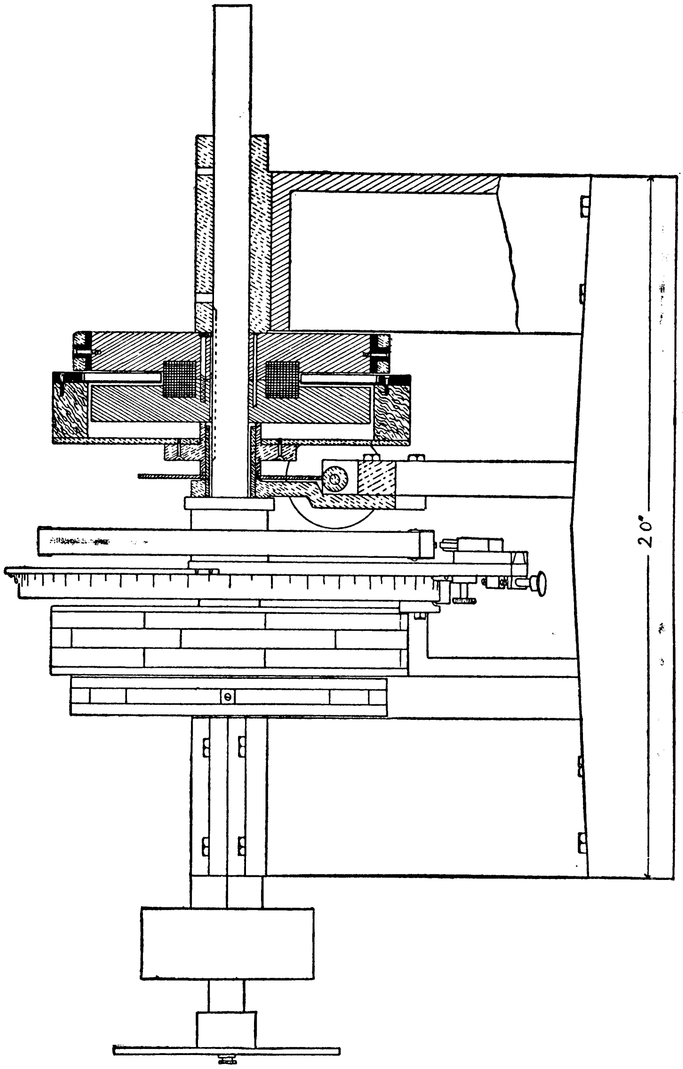

Fig 2 Elevation and Half Section through Dynamo A

Variations of speed are obtained by the use of pulleys of different sizes on the motor shaft.

Both machine and motor rest on a brick pier, the machine being set in sulphur; mechanically, the arrangement leaves nothing to be desired. It has been run continuously at the rate of 3,000 revolutions per minute, without perceptible jarring.

The machine was at first supplied with a mechanical counter, which could be thrown on or off by means of an electro-magnet operated by a key. We found, however, that the arrangement was unsatisfactory, as variations of speed were introduced by its friction and inertia. Moreover, such an arrangement gives only the average speed during a given time; we therefore abandoned this counter, and adopted a stroboscopic device. A ray of light is reflected upon the revolving segmented disk from a mirror attached to an electrically driven tuning-fork of known rate. With such an arrangement the desired speed may be attained, and variations from it measured by noting the apparent motion of the disk.

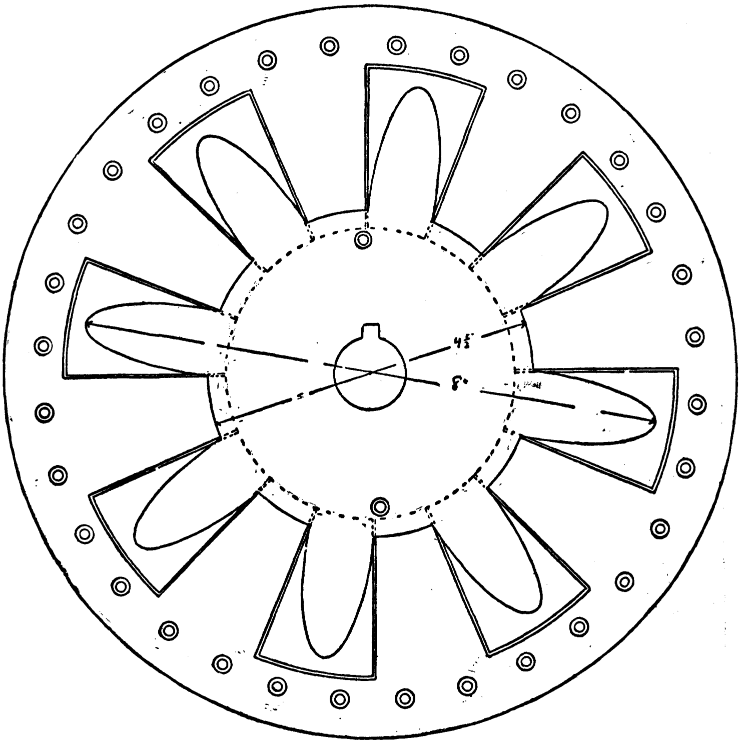

Fig. 3 Field Magnet and Armature

The first test to which the apparatus was put was to obtain the judgment of a person having a trained ear as to the quality of the tone produced by its current in a telephone. His opinion was that the tone was pure, only the fundamental being detected. The wave form was then mapped.

In planning this work it occurred to us to employ the method of projection of potential whenever it was necessary to measure a P. D. or E. M. F. at any instant. The opposing P. D. was obtained by the fall of potential due to the passage of a direct current through a suitable resistance. As the opposing P. D. was not alternating, the above mentioned contact arrangement was used in place of the ordinary key, and its value was determined by a suitable Weston voltmeter. This method of dealing with instantaneous E. M. F.'s was originally suggested by Professors Foster and Maxwell, their object

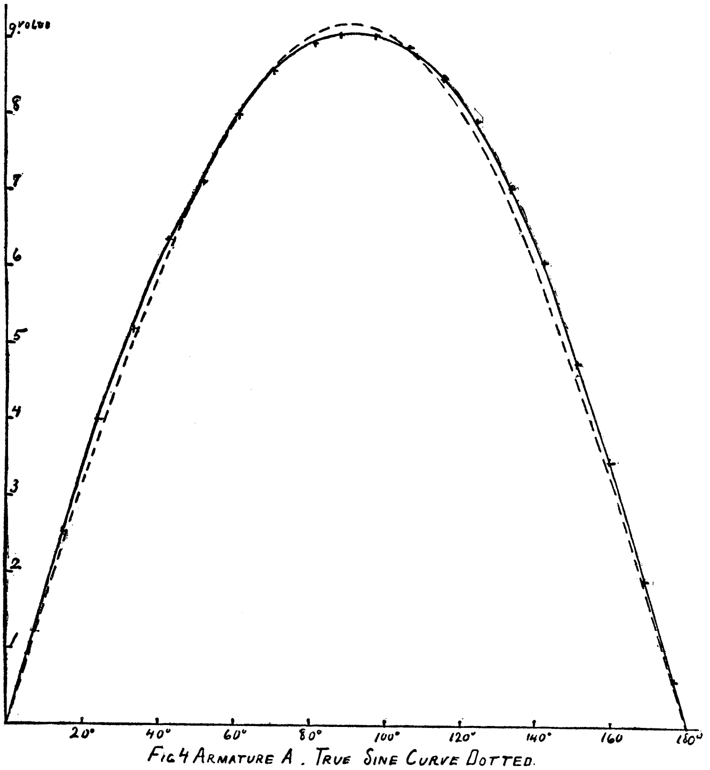

Fig. 4 Armature A. True Sine Curve Dotted.

being to obtain a balance between a known E. M. F. and the P. D. between the terminals of a resistance through which a current of known absolute magnitude was flowing.*

In order to obtain sensitiveness at the final balancing, a condenser one microfarad in capacity was inserted in shunt with the galvanometer, the discharge of the condenser serving to increase the deflection. The galvanometer was of 3,000 ohms' resistance. A telephone was used to obtain the preliminary balance. In Table I. are given the results of the determination of the wave forms of both dynamos, one section of 10 wires being used. In the case of armature A (movable) a series of checks is appended. The whole series was gone through with and then repeated in the reverse direction, the person making

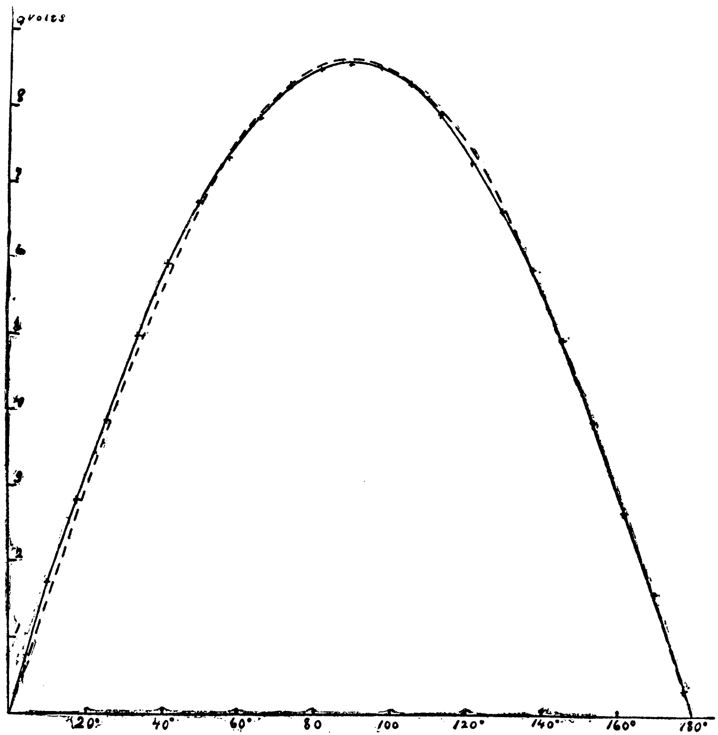

Fig 5 Armature B. True Sine Curve Dotted.

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* Telegraphic Journal, 1874, II. 317.

TABLE I. — DETERMINATION OF WAVE FORM.

ARMATURE A. Ampere Turns, 2,875. — Speed, 1,465.			ARMATURE B. Ampere Turns, 2,150.	
Revs. of Mic. Head.	E. M. F. in Volts.	Check.	Position of Contact.	E. M. F. in Volts.
0	+0.49	+0.48	11°	+1.00
½	—0.91	—1.20	10	—0.26
1	—2.52	—2.53	9	—1.72
1½	—4.01	—4.01	8	—2.80
2	—5.21	—5.19	7	—3.86
2½	—6.38	—6.36	6	—4.98
3	—7.09	—7.21	5	—5.93
3½	—8.04	—8.03	4	—6.75
4	—8.61	—8.61	3	—7.34
4½	—9.01	—8.96	2	—7.86
5	—9.11	—9.12	1	—8.30
5½	—9.12	—9.12	360	—8.50
6	—8.95	—8.94	359	—8.56
6½	—8.52	—8.54	358	—8.54
7	—8.00	—7.95	357	—8.30
7½	—7.09	—7.08	356	—7.91
8	—6.11	—6.12	355	—7.26
8½	—4.80	—4.80	354	—6.62
9	—3.53	—3.48	353	—5.86
9½	—1.94	—1.96	352	—4.93
10	—0.60	—0.60	351	—3.85
10½	+1.12	—	350	—2.67
—	—	—	349	—1.60
—	—	—	348	—0.32
—	—	—	347	+0.74

the readings not knowing the previous results. The most probable sources of deviation are backlash, variations of field current, and change of speed. The checks show sufficient precision of the apparatus for the method of procedure to be described presently.

These results are plotted together with true sine curves in Figures 4 and 5. The curves proved to be very nearly symmetrical, so that only one half has been given. The saturation curve connecting maximum E. M. F. and field current was taken with armature A. The results are given below, and are plotted in Figure 6.

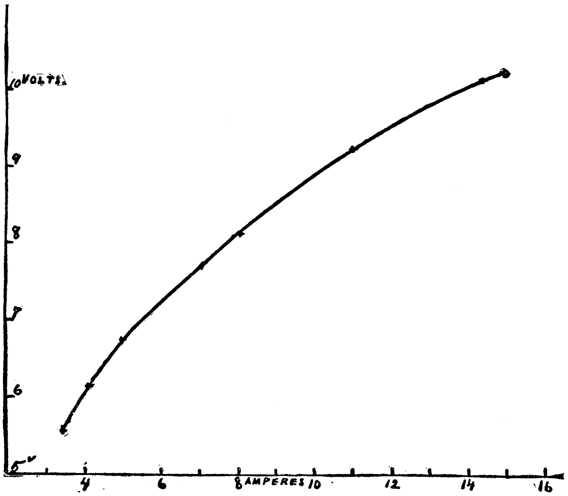

FIG. 6 SATURATION CURVE

TABLE II. — DATA OF SATURATION CURVE.

Speed, 1,465.

Field Current in Amperes.	E. M. F. in Volts.	Field Current in Amperes.	E. M. F. in Volts.
3.40	5.56	8.06	8.10
4.12	6.16	9.38	8.52
4.93	6.74	11.12	9.23
6.10	7.40	14.40	10.10
7.08	7.70	15.00	10.18

METHODS OF USING THE MACHINE.

The following methods are all based on the assumption that we have a sinusoidal E. M. F. of known period, acting in a circuit containing only true resistances and inductances. The current at any instant will be expressed by the equation

$$i = \frac{E}{\sqrt{R^2 + P^2L^2}} \sin\left(Pt - \tan^{-1}\frac{LP}{R}\right).$$

The maximum value of the sine term being unity, we have

$$\frac{I}{E} = \frac{1}{\sqrt{R^2 + P^2L^2}},$$

I and E being the maximum current in, and maximum E. M. F. impressed upon, the part of the circuit under discussion. Also the lag between current and E. M. F. is expressed by $\tan\theta = \frac{LP}{R}$. From these two equations we have

$$L = \frac{E}{IP}\sin\theta, \quad \text{and} \quad R = \frac{E}{I}\cos\theta.$$

Our aim is, therefore, to obtain the values of the impedance, angle of lag, and frequency, and to calculate the coefficient of self-induction and the resistance for any given periodicity by the above formulæ.

(*a*) If we make up a circuit of the movable armature, a known resistance of negligible inductance, and the unknown coil, all in series, we may, by means of the measuring arrangement already mentioned, obtain E and I, the latter by the maximum P. D. around the known resistance. The angle θ would best be found by taking measurements on both sides of the zeros of current and E. M. F., and interpolating for the zero points. The difference of readings of the micrometer so obtained divided by 20 gives the angle θ. L and R are to be calculated by the above formulæ.

(*b*) If we have a carefully made plot, like Figure 6, connecting the maximum E. M. F. and field current of one dynamo, we may proceed as follows. As the wave forms are alike, both following the sinusoidal law, we may apply Poggendorf's method of balancing E. M. F.'s directly, using a suitable telephone instead of a galvanometer. To do this make up a circuit of the stationary armature, known inductionless resistance, and unknown coil. Keep the field current of this dynamo constant. Attach the second armature through the telephone, first to the terminals of the known resistance. Adjust the phase relation for a minimum of sound in the telephone; then adjust for silence by varying the field current. Read the micrometer head and value of exciting current. Repeat this process at the terminals of the unknown coil. These measurements furnish the data necessary for the calculation of L and R. θ is given by difference of micrometer readings divided by 20, as before. Owing to lack of exact similarity of E. M. F. waves it would be necessary in practice to adjust by a minimum, annulling the fundamental.

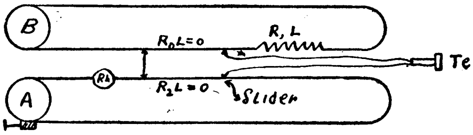

FIG. 7.

(*c*) The trouble of taking the preliminary curve, as well as any uncertainty which its use may introduce, can be avoided by the connections shown in Figure 7.

This method differs from the preceding only in the use of a derived E. M. F. for balancing, and in the method of its adjustment. For convenience give R_1 such a value that the P. D.'s around R_1 and R shall be about equal. Adjust the phase of Armature A by the tangent screw, and adjust for amplitude by varying the field current, final adjustment being obtained by the slider.

Now change the connection to the terminals of the unknown coil, and again adjust for silence, making the adjustment for amplitude in this case by the slider alone. If the two values of the slider resistance are R_2 and R_3, then $\frac{I}{E} = \frac{R_2}{R_1 R_3}$. θ is given by the micrometer head as before.

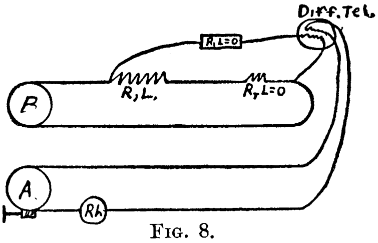

FIG. 8.

(*d*) We may use a differential telephone, that is, a telephone having two opposed windings of equal influence and resistance, in the manner shown in Figure 8.

R_{T} is a resistance equal to that of one coil of the telephone and of negligible inductance. Adjustment is to be made until with the same value of i_{A} the telephone is silent when in place, as shown, and when interchanged with R_{T} then $R_1 = \frac{E}{I}$. θ is given by micrometer readings as before.

(*e*) We may also use the differential telephone to indicate when an unknown inductance is neutralized by a condenser of known variable capacity, by placing one of its coils in series with the inductance and condenser, and the other in series with a variable inductionless resistance R_1, shunted around the above condenser, inductance, and telephone coil, if the resistance R_1 and capacity C be varied until silence is obtained. Then $L = \frac{1}{CP^2}$ and $R = R_1$. It will be desirable to employ an air-condenser to eliminate absorption effects, though this procedure has the disadvantage of requiring a very bulky and costly condenser when small inductances are to be measured.

With this apparatus we may also determine the electrostatic capacity of a condenser when not complicated by absorption effects. For instance, by the first method, we should insert in series with the condenser an inductionless resistance of such a magnitude that the angle of advance could be determined with accuracy.

Then

$$\frac{I}{E} = \frac{1}{\sqrt{R^2 + \frac{1}{C^2P^2}}}, \qquad \tan\psi = \frac{1}{CRP},$$

$$C = \frac{I}{EP\sin\psi}, \quad \text{and} \quad R = \frac{E}{I}\cos\psi.$$

If $R = 0$, $\psi = 90^\circ$ and $C = \frac{I}{EP}$.

The above are the chief uses to which the apparatus may be put. We hope in the near future to investigate the transmission of telephonic and telegraphic signals, and to study various small induction coils such as are used in telephony, as well as the errors introduced into results by the above methods owing to lack of fulfilment of the assumed law by the E. M. F.

Rogers Laboratory of Physics,
September, 1893.